Microwave Resonance in a Waveguide System

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Abstract

A waveguide system in the microwave X-band (~8.5 GHz) is described. A brief introduction to the measurement apparatus is covered. The power, center frequency, and frequency modulation of the source are characterized. The system is tuned to resonance, its $\vec{E}$ field is mapped, and waveguide coupling impedance are explored. Finally, a balanced mixer is used to find the phase shift of reflected microwaves, and a circuit for a Phase-Locked Loop is posited.

INTRODUCTION

Microwave waveguides are used in the transmission of signals and power in broadcast, radar, and other GHz-range RF systems. Waveguides were first invented and used during the investigations into radar during the 1940s, especially at the “Rad Lab” at MIT, among other places.[1]

A waveguide is a hollow tube of a highly conductive metal, which provides a means of directing RF-energy photons. Two views can be taken as regards the mechanism through which this is achieved. One view is that RF photons behave like particles, bouncing off the inside of this “pipe” in various patterns. Another view is that, instead, the photons are behaving like waves, and are being conducted down the length of the waveguide along the walls. Due to the skin effect, and its depth (or lack thereof) at the frequencies of interest in the microwave range, a waveguide of a sufficiently low resistance need not be very thick at all to effectively shield the enclosed cavity, such that it doesn’t leak any energy in the form of radiated RF.[1, 2]

Resonant cavities are very useful in the filtering of microwaves, and are also useful for measuring the frequency of incident RF.[3] This is due to the fact that a cavity will only be resonant over a very narrow band of frequencies. The bandwidth and resonant frequency is dependent entirely on the physical size of the cavity, given that it is made of a high-conductivity material. In one case which is actually used in the experiments described in this paper, a frequency meter is made from two concentric, anti-parallel cylindrical shells. Both of these are threaded, such that one can be fed into and out of the other. Ports allow microwaves in and out of the resonant cavity formed by this structure. When the cavity is tuned to resonance with the incident microwaves, its transmitted power drops. If the downstream power is measured, and if the cavity has been calibrated, it is then easy to create a frequency meter from this device.

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THEORETICAL BACKGROUND

Propagation of electromagnetic waves is governed by Maxwell’s equations. In the case of a rectangular waveguide, such as is considered here, the electric field is oriented in the $y$ direction, given that the waveguide is $\lambda/4$ in width ($a$ in the $x$ direction) and $\lambda/8$ in height ($b$ in the $y$ direction), and that the $z$ axis is parallel to the direction of propagation. Thus, we have the three equations[2]:

$$E_y = E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d}$$

$$H_x = -j \frac{E_0 \lambda}{\eta 2d} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d}$$

$$H_z = j \frac{E_0 \lambda}{\eta 2a} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d}$$

These describe the electric ($E$) and magnetic ($H$) fields within the waveguide. The waveguide itself is rectangular, and is being referred to by these variables as seen in Fig. 1. Thus, when the electric field within the waveguide is measured, it should be shown to behave as a function of the family of $\sin(x) \times \sin(z)$.

Since the waves within the waveguide are standing waves, due to reflection within the waveguide, it is also possible to calculate what their forward to reflected component ratio is. This is known as the Standing Wave Ratio, or SWR, and when measured via the electric field, it is the Voltage Standing Wave Ratio, or VSWR. The VSWR allows a very easy way of mapping the $\vec{E}$ field, and also of finding out the impedance of components in the waveguide.

The power output by the microwave source is also measured, in this case by a thermistor mount. When power is transmitted into this mount, its internal temperature changes, due to RF absorptive material inside it. This $\Delta T$ is proportional to the power input, and so can be used to measure the amount of energy over a given integration time. This will be used to determine the impedance of several parts of the system, as well as to determine efficiency, or $Q$, of power transmission, and also, with the frequency meter, the frequency of the system.

As seen in Fig. 3(A), the system has an equivalent circuit expressed as a series of transmission lines.[2] Going through this from left to right, there is first the microwave oscillator itself, which is frequency modulated. This is approximately equivalent to having an RF oscillator and an LF oscillator (like a function generator) connected to the two input ports of an ideal mixer. The output from the mixer is then attenuated, and transmitted through an ideal transformer to the next stage, equivalent to being isolated by the isolator. The wavemeter acts like a tunable LC circuit, where $L$ and $C$ are linked, and able to be tuned to resonance. The directional coupler is then linked, as if it were another ideal transformer, and has pickups both upstream and downstream to receive forward and reflected power. The slotted line is another segment, once again behaving like an ideal transformer, and passing RF through the iris. The iris acts like an impedance matching circuit[1, 4] (denoted here only as a resistor), and transmits RF out to the perforated waveguide, field probe, and adjustable short, which is herein denoted as a variable resistor. The field probe is effectively an unbalanced output which can be measured against ground to determine the $\vec{E}$ field inside the waveguide.
Finally, the last portion of this experiment involves the creation of a Phase-Locked Loop, or PLL. In a PLL, a phase detector is coupled to a frequency-agile RF source, so that if the phase is determined to have lagged or led, the frequency is adjusted to compensate.[3]

**DESCRIPTION OF APPARATUS**

The apparatus used included a Resotech X-Band solid state microwave oscillator, a 3 dB waveguide attenuator, a ferrite isolator, a mechanical frequency meter, a bi-directional coupler, a slotted line section, a perforated section, a waveguide probe, and a sliding short with micrometer mount. In the first part of the experiment, the power, orientation of the bi-directional coupler, and frequency of the source were determined. The power of the microwave source was determined by connecting the thermistor mount of the power meter to the end of the slotted line section. The frequency was determined by adjusting the mechanical frequency meter until a drop in power was observed on the power meter. When this was observed, the frequency on the meter was recorded. Finally, the power meter thermistor mount was attached to each of the two directional ports on the bi-directional coupler as either a short plate or a high-impedance terminator was attached to the end of the waveguide.

For the next part of the experiment, the perforated waveguide section, waveguide probe, and adjustable short were once again connected to the end of the slotted line section. A sawtooth signal at 60 Hz, 4 Vp-p, was fed into the FM input port of the microwave oscillator, and, through a BNC T-connector, to channel 1 (the \(x\) channel) of an oscilloscope. A diode field probe was connected to the waveguide probe section, and its signal was fed into channel 2 (the \(y\) channel) of an oscilloscope. Thus, the resonance response of the cavity formed by the perforated waveguide section and the adjustable short could be mapped as a function of the displacement of the short. The resonance curve was also viewed in absorption by attaching the diode probe to the reflected power port of the bi-directional coupler.

In the third part of the experiment, impedance matching was explored by attaching different irises in between the end of the slotted line section and the front of the perforated waveguide section. The crystal detector on the slotted line section was connected to an SWR meter, and the SWR of the slot-
Figure 3. (A) physical layout of waveguide experimental system, and (B) equivalent transmission line circuit diagram.

The intensity of the $\vec{E}$ field was mapped throughout the perforated waveguide section. This was done by inserting a piece of sapphire rod into each hole in this section, and measuring the resonant frequency and reflected power resultant from the $\vec{E}$ field perturbation.

In the last part of the experiment, the phase difference between the incident and reflected RF wavefronts was measured. As seen in Fig. 4(A), the output from the field probe was mixed with the split RF input, and the resultant DC voltage was measured. The DC voltage was then coupled into the FM input of the microwave oscillator to create a Phase-Locked Loop, by combining a frequency agile element with a phase-sensing element, as shown in Fig. 4(C). Another variant that was tried was using the forward and reflected power ports on the directional coupler.

Figure 4. Phase difference sensing circuits (A and B) and PLL (C).

as seen in Fig. 4(B).

**DATA**

See Fig. 5.
Figure 5. (A) Wavemeter power data for various tunings around the RF source frequency. (B) DC level effects on frequency through the FM input. (C and D) $\vec{E}$ field mapping at 8.4550 GHz, short at resonance at 18.15 cm, with a sapphire rod of .125 cm $\phi$, on z and x axes of the perforated waveguide. (E) Iris diameter effect on resonant frequency. (F) Phase shift measured in mV as a function of resonator short depth. This is measured from the IF output of the mixer.
ANALYSIS

The data in Fig. 5(A) clearly show that the frequency of the microwave source is closest to 8.4455 GHz. This was the result gathered while the FM input was shorted, and thus at 0 VDC. It would then make sense that this is the fundamental frequency of the microwave oscillator. Furthermore, the data also clearly suggest that the oscillator is relatively narrow-band. This is due to the $\Delta f$ being only from 8.4440 GHz to 8.4470 GHz.

The DC input levels to the FM input and their correlation to the output frequency of the microwave oscillator are not quite linear, as seen in Fig. 5(B). In point of fact, the fit is actually slightly cubic. This may be due to some adjustment in the oscillator that has drifted from its initial calibrated value. Then again, the original set frequency when the oscillator left the factory was a center frequency of 8.500204 GHz at 28 °C. This was measured at 21.7°C, some 6.3 °C cooler, and twenty years later. The aging drift of any linear oscillator can be modeled, and this one fits within a reasonable range for the time that it has been drifting for.[5] Also, drift of potentiometers is a nearly unknown topic in aging studies, but one might assume that, since a trimming potentiometer (colloquially known as a trimpot) has a pendular arm, it might tend towards a position of least potential energy, thus offsetting the $R$ value originally set. Thus, the gain and offset settings may have become out of alignment over the last twenty years.

The $P_{REF}$ values found from the perforated waveguide and sapphire rod in the $z$ direction, as seen in Fig. 5(C), are congruent with the expectation put forth by Ramo et al.[2] that:

$$\vec{E} \approx \sin x$$

Thus, when it is seen that the reflected power output is sinusoidal in nature, it can be assumed that the fit will also be sinusoidal, as seen in Fig. 6.

![Figure 6. The fit curve for the reflected power in the perforated waveguide.](image)

The reflected power seen across the $x$ axis in Fig. 5(D) is also in keeping with the distributions that are expected, as per Ragan.[1] In this case, the $\vec{E}$ field is a half-wave of a sinusoid in its form.

The resonant frequencies of irises seen in Fig. 5(E) have a certain trend to them. At a short, or 0 cm gap, the resonant frequency is quite low. In fact, it is outside of the bandwidth of the microwave oscillator at shorted FM input entirely. It is only by introducing a swept signal from -4 to 4 V that one can ascertain this value. The rest of the values ramp up to the highest resonant frequency, and then come back into the range of the center frequency of the microwave oscillator with grounded input.

Finally, the phase shift seen in Fig. 5(F) is commensurate with the approach to, and passing of, the resonant point in the micrometer’s travel for the
short in the waveguide. The phase shift should be of $2\pi$ when at resonance, and when constructive interference is at its maximum. Just slightly past this, the anti-resonant trough is observed. Given that the wavelength is 3.55 cm, then the phase shift seen in the figure will repeat every $\lambda$, and it will remain well outside of the scope of this graph.

**UNCERTAINTY**

The uncertainty of this system is dependent on three different orders of uncertainty. The first order involves direct observation of the units on mechanical devices. The second order involves numerical uncertainties where electronic displays are concerned, along with aging characteristics. The third involves environmental variables.

In the first order, the devices that create the uncertainty limits are those with direct, mechanical readouts. The wavemeter and micrometer on the resonator are two great contributors to the uncertainty. The wavemeter has a maximum resolution such that one can tell when the indicator line is directly between two tics of the meter. However, this places the resolution at .5 MHz, with an uncertainty of $\pm .25$ MHz. The micrometer is a whole different case, as it has a resolution of .0005 inches. This places its uncertainty at .00025 inches, but the calculated uncertainty due to rounding from the conversion to cm makes both of these numbers unduly higher.

The VSWR and power meters are also mechanical in their outputs, and are true gage meters. The VSWR meter, which is in dB, is further complicated by the fact that its uncertainty and resolution are logarithmically defined. The power meter is significantly easier to characterize, as it has an uncertainty of .05 mW, and a resolution of .1 mW in the linear range that was used in this experiment. An interesting problem was that during the first week or two of the experiment, the power meter had a loose thermistor mount cable, which, when touched, would vary the reading by as much as .25 mW. This was fixed before any data that were used in this paper were taken.

The digital multimeter (DMM) that was used has a simple uncertainty of .1 mV, but would usually need at least a minute to settle at a specific reading. In the case where the DMM was changing rapidly, an averaging technique over a longer period of time was used to come up with a coherent reading. The oscilloscope, as a relative measurement device, didn’t introduce any real uncertainty, as its readings were double-checked with the DMM.

At any time, the problem of aging could be seen manifesting, as the DMM had a voltage offset of .02 mV in the DC 200 mV range, and the microwave oscillator which has drifted by over 50 MHz since its acquisition in 1991. As previously discussed, the slip of potentiometers may have a great deal to do with this, along with thermal expansion and contraction over time.

The lab was always between 21 and 22 °C, and had a barometric pressure that ranged from 990 to 1005 mBar over the month during which the experiment was being performed. The humidity of the air was not measured, due to a lack of proper equipment. This is troubling, as it has the greatest effect on the dielectric constant of air, which has due relevance to
the operation of microwave waveguides.

Finally, all electronic devices were allowed a warm-up and stabilization period of at least a half hour before any relevant data was taken.

**CONCLUSION**

A series of measurements were carried out on a microwave waveguide system. These included measurement of the system’s emitted power, center frequency, resonant frequency, and coupling impedances. The center frequency was found to be 8.4455±0.0025 GHz. The emitted power was found to be 0.46±0.01 mW. The resonator was found to work best at the center frequency with a setting of 0.623±0.005 inches. Finally, the .9 cm iris was found to have the best coupling efficiency and $Q$ of any of the irises used.

This experiment builds on many of the basic elements of RF and microwave theory that were discovered and invented in the 1940s during the invention of radar. There are a great many advancements that were made in this field using not much more, and in some cases much less, than what was used in this experiment.[1]