$1/f$ and Johnson-Nyquist Noise in metal-film and carbon resistors

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Abstract

Johnson-Nyquist and $1/f$ noise are observed at various temperatures in metal-film and carbon resistors. The noise types are separated in this way by thermometry, and are quantified by power over a low-frequency band, and from ~ 350 K to 4.7 K. The Johnson-Nyquist noise is found to be temperature-dependent, while the $1/f$ noise is not.

INTRODUCTION

Noise is inherent in just about every physical system. In electronics, some amount of noise is always present in every circuit. This noise is necessarily stochastic and unpredictable in nature. However if the band and power of the noise might be predicted, then care can be taken to avoid selecting this band for use in transmitting a signal.

The two types of noise that were explored in this work were "flicker", or $1/f$, and Johnson-Nyquist, or thermal, noise. $1/f$ noise is a natural phenomenon which can be expressed as the proportionality of the Power Spectral Density (PSD) to the frequency at low frequencies. Johnson-Nyquist noise is another natural phenomenon which is related to the temperature and resistance of a conductor, and which results from electron excitation due to thermal equilibrium with the surroundings.

Johnson-Nyquist noise can be used to determine temperature or the Boltzman constant, k_B, given that the other of these two quantities and the resistance are well known[3]. This makes it an excellent characteristic to measure in situations where the temperature is too low to be taken by the relatively normal means of a thermocouple or thermistor. This very method has been used for the measurement of temperatures of Bose-Einstein condensates[4]. Similarly, in cases where the temperature is well known, the method has been used to determine the Boltzman constant to a very high accuracy.

$1/f$ noise is found in many materials and systems, such as resistors, solid-state

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semiconductor devices[5], such as CMOS RF mixers[6], oscillators[7], vacuum tubes[5, 8], networks[9], India ink[10], and many others. While its source is not well known, it appears to be non-thermal. Milotti has proposed that the PSD of this noise is related to the diffusion or transport eigenvalues of a white (Johnson-Nyquist) noise driven system[11].

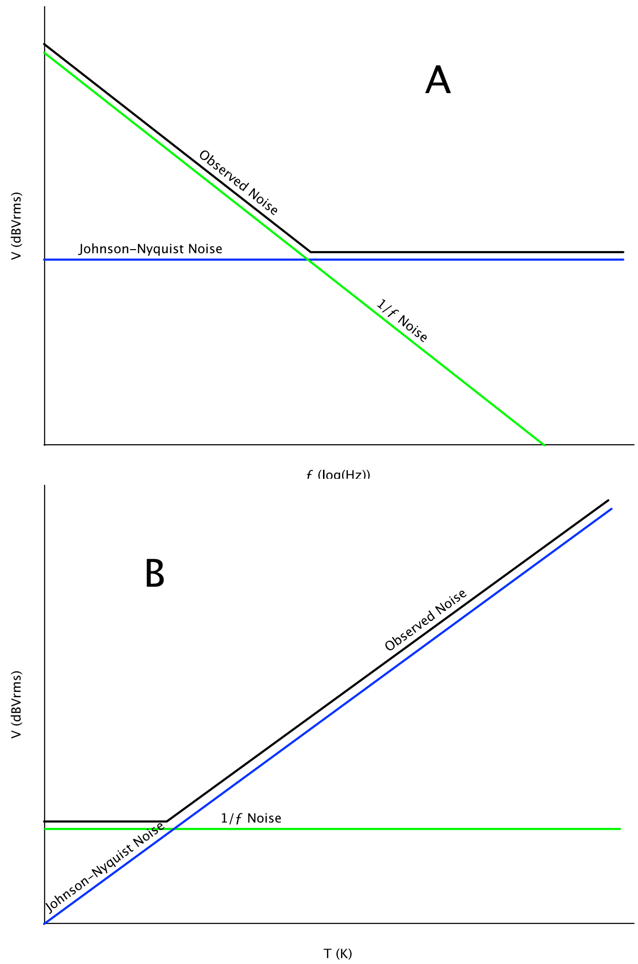

Figure 1. The first graph (A) is a logarithmic plot of the $1/f$ noise and Johnson-Nyquist noise components of the observed noise, which is the sum of the two. The second graph (B) shows the relative intensities of the Johnson-Nyquist and $1/f$ noises as a function of temperature.

THEORETICAL BACKGROUND

Johnson-Nyquist noise is described as being "white", which means that it has a uniform PSD across its entire spectrum, while $1/f$ noise is described as being "pink", meaning that it has a higher PSD at lower frequencies[12]. As seen in Fig. 1, both Johnson-Nyquist and $1/f$ noise contribute to the PSD measured (in dBVrms). At lower frequencies, the $V_{1/f}$ component is larger, while at higher frequencies, the V_{J-N} component is higher. The corner frequency is determined by the temperature, and the overall noise is given by the expression:

$$V_{observed} = V_{J-N}(f) + V_{1/f}(f)$$

The relationship between the Johnson-Nyquist noise, temperature, resistance, and the Boltzman constant, k_B is very well defined[4, 13, 14]. Given a temperature, T, in Kelvins, a resistance, R, in Ohms, a bandwidth of Δf, and an RMS voltage of $\bar{V}$, the relationship is:

$$\bar{V} = \sqrt{4k_B T R \Delta f}$$

[3, 8]

This also gives k_B as:

$$\frac{\bar{V}^2}{4TR\Delta f} = k_B$$

DESCRIPTION OF APPARATUS

The apparatus was set up in two different configurations: in one, the measurements were done as PSD plots and band calculations on the HP 35660A Signal Analyzer; in the other, the measurements were done through an EG&G 7220 DSP lock-in amplifier, and data was taken on a computer.

Before any PSD measurements were done, the resistance of each of the samples was determined at a series of temperatures ranging from 4.7 to 350 K. The samples were carbon and metal-film resistors in a sample stage, as shown in Fig. 2. The quiescent currents and voltages across each resistor were also measured.

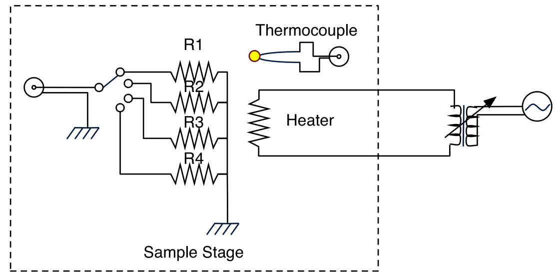

Figure 2. The sample stage contains all four sample resistors. R1 and R3 are metal-film resistors, while R2 and R4 are carbon resistors. The BNC on the left side of the sample stage is the means for output to any sensing device, while the heater on the right side is connected to a Variac to control temperature in the stage. A K-type thermocouple is used to measure the temperature for any setting above 150 K.

In the PSD measurements, the sample stage was connected to a bias box, as in Fig. 3. The bias box was first connected to an ammeter to measure the current across each resistor (and the bias bridge in the box), then the ammeter connection was disconnected and shorted, and the V_{sense} connector was connected to a PAR 113 pre-amplifier. The amplifier was then connected to the HP 35660A, and set so that its filters didn't interfere with the measurement of noise across several bands. In this way, the combination of $1/f$ and Johnson-Nyquist noise can be measured. This is done at a series of temperatures at or above room temperature using the heater. The sample stage is then inserted into a dewar of liquid nitrogen. After the measurements are done in liquid nitrogen, the sample stage is loaded into a tank of liquid helium.

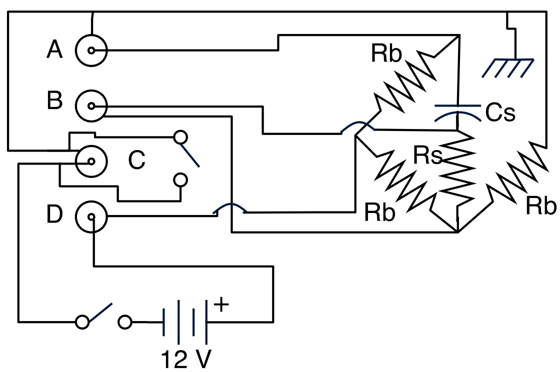

Figure 3. The bias box is mainly a Wheatstone bridge with a shunt in the middle. The sample is connected to connector A, while the V_{sense} output is on connector B, the ammeter output and short are on C, and the external variable resistor is connected to connector D. R_b is 100 Ω, while R_s is 10 kΩ, and C_s is 22 pF.

Another sample, this one designed for use with the lock-in amplifier, as seen in Fig. 4, is connected to the ADC and DAC of the lock-in amplifier. A DC current-voltage measurement is done first, then another is done with the PAR 113 in between the ADC and the sample. Finally, an AC measurement is done with the A input and the Osc output of the lock-in.

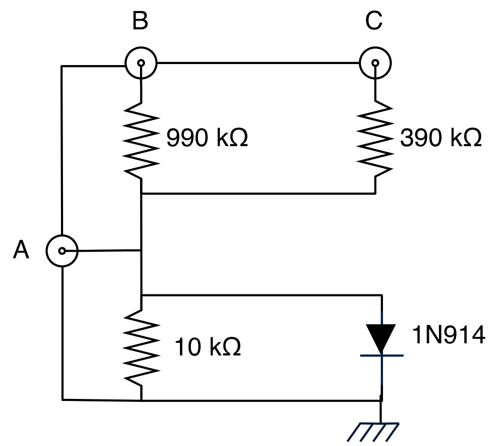

Figure 4. The sample for use with the lock-in amplifier is comprised of three resistors and a diode in an H-bridge configuration. In the DC measurements, the A connector is the output, which is connected to the ADC1 connector on the lock-in in the first test, and the input of the PAR 113 in the second test. In the AC measurement, the A connector is connected to the A input on the lock-in. In the DC measurements, connector B is not connected, and connector C is connected to the DAC1 output connector on the lock-in. In the AC measurement, connector B is connected to the Osc output on the lock-in, while connector C remains connected to DAC1.

DATA

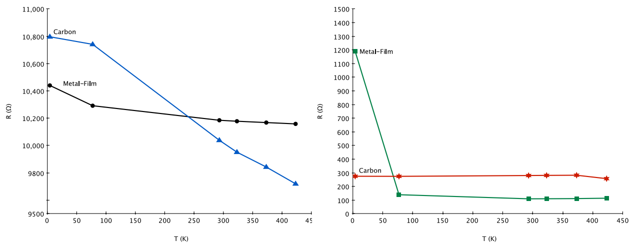

Figure 5. The resistor values for the sample resistors at various temperatures. The uncertainties are small enough (.001 Ω) that the error bars are not shown.

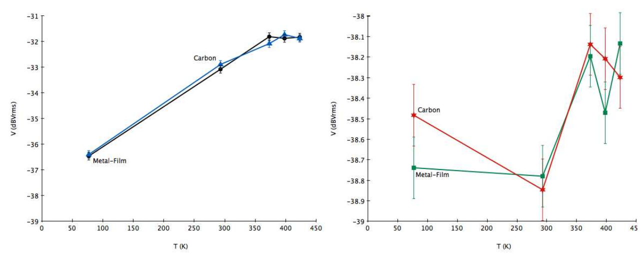

Figure 6. The measured values of $\bar{V}$ at various temperatures. The bandwidth, Δf, is 25 kHz.

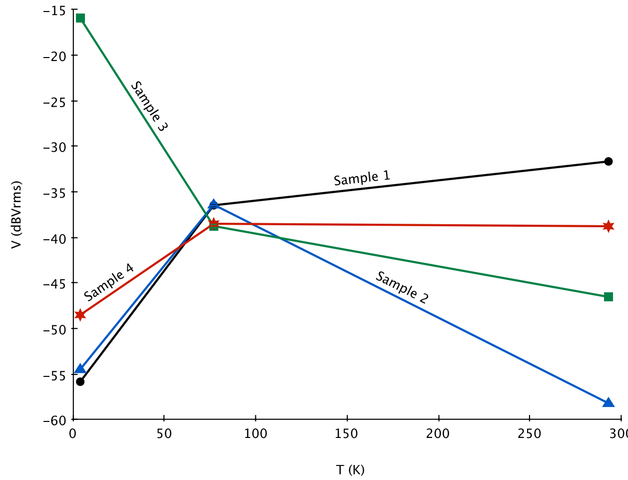

Figure 7. The measured values of $\bar{V}$ at various temperatures. The bandwidth, Δf, is 25 Hz.

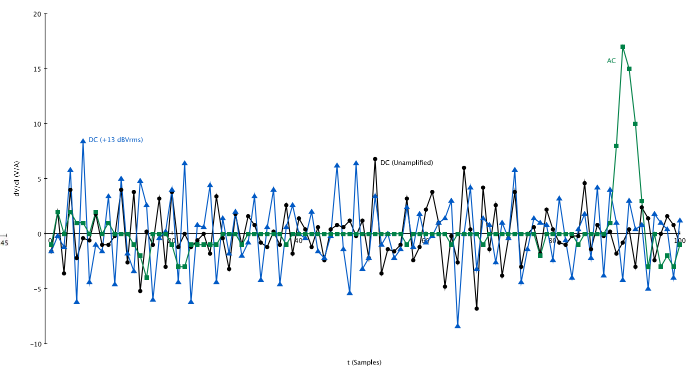

Figure 8. Data from the Lock-In amplifier experiment as $\frac{dV}{dI}$ as a function of time.

ANALYSIS

First, to the Boltzman constant calculation. Given that, as stated above, $\bar{V} = \sqrt{4k_BTR\Delta f}$, and so the expected values of $\bar{V}$ can be calculated for a known $R(T)$. This gives us what we see in Fig. 10. However, these numbers, while close to what is expected, are all slightly higher. This could be due to the fact that this calculation was done with a sufficient bandwidth that the $1/f$ noise could still be adding a significant amount of power to the noise PSD. Also, there could have been significant other noise sources in the lab. Finally, the PAR 113 amplifier may not be completely linear, which would also lead to a possible shift over different voltages.

From the above formula, k_B can also be calculated. In this case, the average value

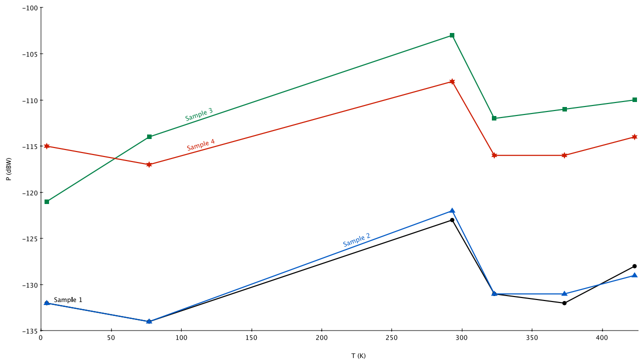

Figure 9. The quiescent power ($1/f$ and Johnson-Nyquist combined) as a function of temperature.

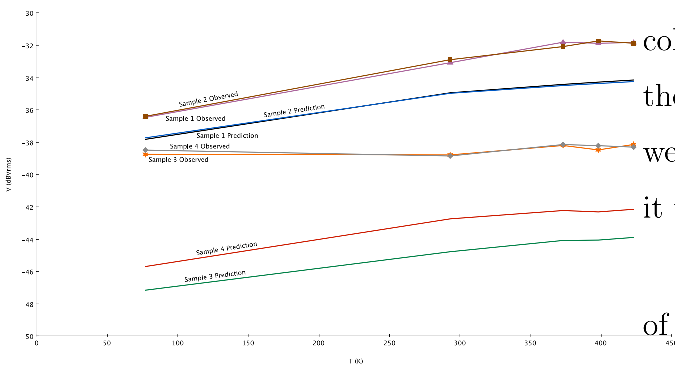

Figure 10. The discontinuous predictions and observations of $\bar{V}$ for the four samples.

was around 3.2(±.1) aJ/K, still very far off. Given the uncertainties, and the observed noise amplitudes, this is to be expected. Also, the two higher resistance samples, as seen in Fig. 11, actually encompass the true value of k_B in their measurement uncertainties.

Another odd aspect to this experiment was the strange reading on the resistance of Sample 3 at 4 K. One reason that this could have gone from being a 100 Ω range to 1 kΩ range

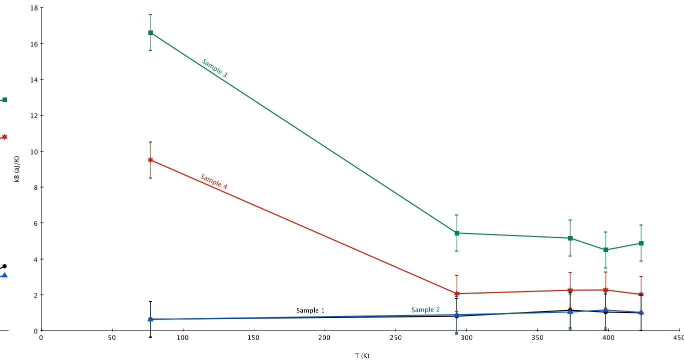

Figure 11. The calculated values of k_B from $R(T)$ and $\bar{V}$.

resistor could be from contraction due to the colder temperatures. As seen in Fig. 12, the strain of thermal contraction could very well have distorted the metal film such that it would have increased its resistance.

In Fig. 6, $\bar{V}$ is measured at a bandwidth of 25 kHz at a range of temperatures. This is done between 10 and 35 kHz to hopefully avoid a large contribution by $1/f$ noise, so mainly Johnson-Nyquist noise can be measured. This data was then used to calculate k_B as shown above, as well as to find the overall PSD of Johnson-Nyquist noise in the system.

In Fig. 7, the $1/f$ noise becomes the primary component, and was measured from 25 Hz to 50 Hz. One possible reason for the convergence of the $\bar{V}$ values at 77 K is that this is the temperature at which thermal noise was at its lowest in the small dewar. In the larger liquid He dewar, the sample was isolated from

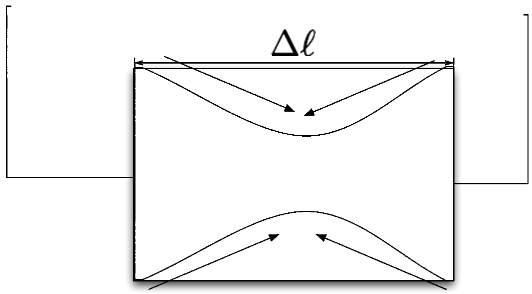

Figure 12. The simplified possible stretching configuration within the metal film resistor. The left and right sides are held rigid on a ceramic mandrel with a smaller thermal contraction tensor than that of the metal film, causing von Mises stresses in the directions indicated by the arrows. By the strain created in the left-right major axis, the length would stay the same, while the cross sectional area of the film would decrease, thus increasing the resistance.

ground, and left electrically floating. In the smaller dewar, the He tank flange was used as a cover, and so a smaller Amperian surface was created, with a lower overall resistance, and a higher efficiency as a Faraday cage. It would then stand to reason that the noise incident on the samples would be higher in the higher-volume dewar with the higher surface area. Another, possibly larger, portion of this divergence is due to the resistance anomaly in sample 3 at 4 K.

In Fig. 8, the $\frac{dV}{dI}$ curves for the DC, amplified DC, and AC lock-in measurements are shown. In essence this is showing the real part of the impedance function, $Z(f)$. If one takes this as $\frac{dV(t)}{dI(t)} = R$, then the real parts of the reactance and impedance can be used to calculate the Boltzman constant, k_B. However, the noise background is the random part of the signal, while there is an actual, periodic portion of the signal which is going through the sample (as in Fig. 4). This is what makes the lock-in ampifier unique: it "knows" what signal it's putting out, and can recover the parts of the signal that are returned to be compared with the outgoing signal. The same could be done with the HP 35660A, though it was only used passively in this experiment.

In Fig. 9, the so-called quiescent power is graphed as a function of temperature. This graph shows an inflection point at about 293 K. This would seem to be caused by either a ranging error in the multimeter, or by a phase change of some sort in the samples or the connectors used with them. Overall, however, the error seems more systematic than tied to a particular piece of equipment.

UNCERTAINTY

The first thing to be considered regarding the uncertainties in this experiment is the mixture of logarithmic and linear scale units that were used. dBVrms were used right alongside Vrms and VDC. Similarly, frequency was often displayed on a logarithmic scale, despite being a linear unit. This leads to a large uncertainty, as one digit's variance can mean as much as an order of magnitude when taken as the exponent to retrieve the linear unit. It would be advisable that, if this experiment were to be done again, only linear units be used.

Efforts were undertaken to understand the behavior of the instrumentation before the experiment commenced. The multimeter was verified using a variety of resistors, and the HP 35660A was verified using a function generator and an oscilloscope. However, these are only transfer standards. No external standards were used in this experiment, which leads to a lack of traceability and verifiability in the equipment and the data taken with it.

It is also true that the air pressure and temperature were known throughout the course of the experiment to be 22(±1) °C and 1.001(±.001) atm, respectively. However, the humidity was not known. Given that these three factors are all part of the calculation of the capacitance of air, and given that the capacitance of air is part of the possible factors in introducing further noise, these should be known. Between all in-air leads, a capacitor could be assumed. Similarly, the loss current and capacitance of the BNC cables used in the experiment should be accounted for. This could factor in as

$$\bar{V_C} = \sqrt{\frac{k_B T}{C}}$$

[15]

Another source of uncertainty is the equipment itself. The thermocouple thermometer has an uncertainty of ±1 °C. The multimeter has an uncertainty of ±.001 Ω in the resistance mode, ±10 µA in the ADC mode, and 1 µV in VDC mode[16]. The HP 35660A has an uncertainty of ±.15 dBVrms, and ±.003% of reading Hz[17]. The EG&G 7220 has an uncertainty of ±20 nV in the voltage circuit, ±20 fA in the current circuit, and a frequency uncertainty of ±.0001 Hz[18].

CONCLUSION

In this experiment, a series of four sample resistors were measured for resistance, quiescent current, Johnson-Nyquist noise, and $1/f$ noise at a variety of temperatures. The Boltzman constant, k_B, was derived and calculated from the PSD power from thermal noise. $1/f$ noise was observed at a variety of resistances and temperatures, and noise thermometry, as previously described, was also performed. A lock-in amplifier was used to try to calculate the non-linear resistance of a mixed solid-state sample. Most of all, the $1/f$ and Johnson-Nyquist noise were discriminated, such that both could be measured with a minimum of interference from the other.

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